

Scientific Project: Singularity formation in the Keller-Segel system

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Proposal: In this project we focus on the study of singularity formation for the classical Keller-Segel system modeling biological chemotaxis processes and stellar dynamics. Singularity formation is important for understanding the physical limitations of the models: at the singularity, the physical validity of the model necessarily breaks down. Even if the singularity is known to occur, understanding the precise dynamics would be important for determining what additional physical effects need to be taken into account for a proper model. Moreover, for models lacking simple monotonicity formulas, proving the singularity formation may simultaneously require a precise description of the dynamics near the singularity. After the works [1, 3, 2], we are interested in investigating new types of singularities such as multiple-collapsing solutions (collision phenomenon) in both L^1 -critical and L^1 -supercritical regimes. A few results that describe the collision of solitons have been established for wave-type equations, see for example [4], [5] [6, 7, 9], [8] from which basic ideas could be used to tackle the Keller-Segel system. This is a joint project with C. Collot, T. Ghoul, N. Masmoudi.

References

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