Scientific project:
Title: Detecting separately inclusions immersed in a fluid by a dynamic mesurement
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The project is part of a general research theme focused on the modeling of flows of in- compressible viscous fluids in and/or around a bounded obstacle. Generally speaking, the aim is to detect small obstacles immersed in a fluid flowing in a known bounded domain? in the two or the three dimensional case. We assume that the fluid motion is governed by the nonstationary Stokes equations. This dynamical case extends those already concerned with the detections of small bounded inclusions by using boundary measurements for solutions of Stokes equations (stationary case). This kind of inverse problems arises, for example, in moulds filling during which small gas bubbles can be created and trapped inside the material or in the detection of mines ...

The detection of objects immersed in a fluid, governed by the (stationary) Stokes equa- tions, has been a subject of active research for many years (see [1, 4, 10, 5, 8, 9]). To extend the previous works to our dynamic detection problem, we make firstly the assumption of small size of the objects to use asymptotic formulae. Based on this asymptotic formulae and layer potential techniques, one can use the idea for stationary case in [2, 3, 7] to develop an inversion algorithm. The numerical algorithm may be based on the coupling of a discontinuous Galerkin method for the time dependent Stokes equations, on the exact controllability method and on a Fourier inversion.

Moreover, as particular but important and ambitious part of this work is to mathematically describe the dynamics of a small deformable droplet shape $D_\epsilon(t)$. We recall the reader that the small immersed droplet can be seen as an inclusion of small size ϵ in the Newtonian fluid domain Ω. The idea here is to derive the asymptotic expansion of the velocity field $v_\epsilon(x,t)$ as the droplet diameter ϵ tends to 0. Thus, by using previous described method and boundary measurements on $\partial\Omega \times [0,T]$ (for $T > 0$) we may detect the main information for $D_\epsilon(t)$ with $t \in [0,T]$.

This research will be carried in collaboration with prof. C. Daveau and his group at the Laboratoire Analyse-Gomtrie-Modlisation (AGM).

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