Scientific project:

<u>Title:</u> A numerical method for reconstructing of small interface changes of an inclusion from modal measurments: The Stokes case

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In this project, we consider the Stokes eigenvalue problem in a bounded domain Ω of $\mathbb{R}^n$, $n = 2,3$ with Dirichlet boundary conditions:

$$-\mathrm{Div}(\sigma_{D_\delta}\mathcal{D}(v_\delta) - p_\delta Id) = \lambda_\delta v_\delta, \quad \nabla.v_\delta = 0 \text{ in } \Omega \text{ and } v_\delta = 0 \text{ in } \partial\Omega.$$

$\mathcal{D}$ means the strain rate tensor and $\sigma_{D_\delta} := \sigma_0\chi_{\Omega\setminus\overline{D}_\delta} + \sigma_1\chi_{D_\delta}$ where the tensors σ_0 and σ_1 may be given by $(\sigma_s)_{ijlk} = \mu_s(\delta_{ki}\delta_{lj} + \delta_{kj}\delta_{li})$ for $i,j,k,l = 1,2$ and $s = 0,1$. Here μ_0 and μ_1 are the viscosity constants of the flow in $\Omega\setminus\overline{D}$ and D, respectively.

In this work we will first derive the leading-order term in the asymptotic formula for the eigenvalue perturbation due to small changes of the interface $\partial D_\delta := \{\tilde{x} = x + \delta h(x)\nu(x), \quad x \in \partial D\}$ in a Newtonian fluid material. Our derivations are rigorous and proved by layer potential techniques. To reconstruct the perturbation ϵh from modal measurements, a first idea is to minimize the difference between the measured and the computed eigenvalues by using a least-square approach. This gives a laborious reconstruction algorithm which may not converge if we start away from the solution.
The inverse problem considered in this project is to recuperate some informations about h from the variations of the modal parameters:

$$(\lambda_0 - \lambda_\delta, \sigma_D[\mathcal{D}(v_0) - \mathcal{D}(v_\delta)]\nu|_{\partial\Omega})$$

associated with the underlined eigenvalue problem. Based on the dual asymptotic formula, we will propose optimization approaches for reconstructing the interface changes from either complete or incomplete data. The method for reconstructing the shape deformation in the case of a simple eigenvalue λ_0 is to minimize the functional $J(h)$ over h where J is given by

$$J(h) := \sum_{l=1}^{L} \Big| \int_{\partial\Omega} g_l \cdot (\mathcal{D}(v_\delta) - \mathcal{D}(v_0))\nu - (\lambda_0 - \lambda_\delta)\int_\Omega w_{g_l} \cdot v_0 - \delta \int_{\partial D} h(x)\mathcal{V}[\mathcal{D}(v_0^e)](x) : \mathcal{D}(w_{g_l}^e)(x)ds(x)\Big|^2,$$

where $g_1, \cdots, g_L$ are linearly independent vector-valued functions such that $g_l \in L^2(\partial\Omega)$.

We will perform numerical experiments to test the viability of the proposed algorithms. We will give different examples such as minimization using significant eigenvectors and incomplete measurements. The background domain Ω is assumed to be the unit disk centered at the origin, the inclusion D is a disk centered at (0,0.1) with the radius 0.5, and we suppose that $(\mathcal{D}(v_\delta) - \mathcal{D}(v_0))\nu$ is measured in $\partial\Omega$.
We believe that our results are ambitious tools for determining the locations and/or shapes of small inhomogeneities (impurities) by taking eigenvalue measurements. This research will be carried in CY Advanced Studies with the professor Christian Daveau from the AGM research laboratory.

<u>References:</u>

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