SCIENTIFIC PROJECT CERGY 2022

Let us set the stage by briefly introducing three correspondences. Each correspondence pertains to an algebraic group defined over a field. The first correspondence is the deep and largely conjectural Langlands Correspondence

$$representations \longleftrightarrow L-parameters.$$

The objects on the left, the automorphic representations, may be thought of as generalizations of classical modular forms. The Langlands Correspondence is also valid over p-adic or real fields, where it still endows the representations with number-theoretic content. We shall restrict our attention to the real field below. We label the second correspondence, unfairly, with harmonic analysis

$$representations \longleftrightarrow classfunctions.$$

For finite groups, this correspondence is the well-known association of a representation with its character, a function on conjugacy classes. We label the third correspondence, again unfairly, with sheaf theory

$$representations \longleftrightarrow sheaves.$$

This correspondence is less well-known, but may ring a bell for those who have heard of Kazhdan-Lusztig polynomials. With the three correspondences in place, we are able to present the main objective of our proposal.

Objectives: A-packets are sets of representations with common number-theoretic features given through the Langlands Correspondence. They are traditionally defined using harmonic analysis, but an alternative definition, introduced by Adams-Barbasch-Vogan, using sheaf theory also exists. In joint work with Adams-Arancibia it is verified that the two definitions coincide for real classical groups. This places sheaf-theoretic methods on equal footing with analytic methods when considering properties of automorphic forms at archimedean places, offering us new perspectives, and tools to solve problems. The objective of this project can be divided in the following topics:

Topic A: Extend the equality in Adams-Arancibia-Mezo, between the two definitions of A-packets for classical groups, to unitary groups.

Topic B: Use the existing ATLAS-software for Lie groups to compute A-packets for classical and unitary groups. A-packets are parameterized by A-parameters, the objective is to implement a function in ATLAS such that by entering an A-parameter, the output would be the corresponding A-packet. Having a tool which computes A-packets should open the door to a better understanding of their properties. Moreover, the use of ATLAS have shown to be useful in the past to make or verify conjectures that ultimately have shown to be true.

Topic C: Implement the sheaf-theoretic methods of Adams-Barbasch-Vogan in answering questions that, until now, had mostly been studied from the harmonic analysis perspective. We would like to use these sheaf-theoretic tech-

niques to determine the structure of the A-packets. This could lead, for example, to show that A-packets for classical groups satisfy the multiplicity one property.

Topic E: Find a connection from the sheaf-theoretic methods in determining whether or not a representation is unitary.